CARINGBAH HIGH SCHOOL

YEAR 12 (2010) ASSESSMENT TASK 1
MATHEMATICS (2 UNIT)
DECEMBER 2009

Time Allowed: **65 minutes**

Instructions:
- ***Start each "Question" on a new page**
- ***Show all necessary working**
- ***Do <u>not</u> use liquid paper**
- ***Silent, nonprogrammable calculators may be used**
- ***Attempt all questions**

Marks

Question 1 (8 marks) (Start a New Page)

a) The nth term of a sequence was given by $T_n = 5 \times 2^{n-1}$. Find T_3. **(1)**

b) Differentiate with respect to x:

(i) $\sqrt{x}$ **(1)**

(ii) $(4x-1)^6$ **(1)**

(iii) $\dfrac{2x+1}{3x-1}$ **(2)**

c) Find the value of 'k' if $x^2 + 3x + k = 0$ has real roots. **(3)**

Question 2 (7marks) **(Start a New Page)** **Marks**

a) Solve the inequality $(x-2)(x+5) \geq 0$ **(1)**

b) Solve for x: $x^4 - 3x^2 - 54 = 0$ **(3)**

c) Sketch an arc of function $y = f(x)$ on a set of coordinate axes, given that: **(2)**

$$f'(x) > 0$$
$$f''(x) < 0$$

d) Using the information in the table below, what type of point is found at $x = 3$ for this function? **(1)**

x	2.9	3	3.1
$\frac{dy}{dx}$	-0.8	0	-0.6

Question 3 (7 marks) **(Start a New Page)**

a) For what value of k does the equation $x^2 + kx - 8 = 0$ have a root of 3? **(2)**

b) If α and β are the roots of the equation $2x^2 + 5x - 1 = 0$ find, without solving the equation, the value of:

(i) $\alpha + \beta$ **(1)**

(ii) $\alpha\beta$ **(1)**

(iii) $\alpha^2 + \beta^2$ **(1)**

c) If $y = 5x^{3.4} - 3x$, find $\frac{d^2y}{dx^2}$ in simplest form. **(2)**

Marks

Question 4 (8 marks) (Start a New Page)

a) If $2x^2+3x+1 \equiv A(x-1)^2+B(x-1)+C$ find the values of A, B and C. **(3)**

b) Evaluate $\sum_{r=1}^{3}(r+5)$ **(2)**

c) For the arithmetic sequence $\{ 5, 8, 11, \ldots \}$

(i) find the general term T_n in its simplest form **(2)**

(ii) find which term is the first to exceed 200 **(1)**

Question 5 (8 marks) (Start a New Page)

For the curve with equation $y=x^3-9x^2+24x+2$

(i) Show that there are stationary points at $x=2$ and $x=4$. **(2)**

(ii) Find the inflexion point **(2)**

(iii) Sketch the curve, including the above information. Use a half page diagram, correctly labelled. Include the y-intercept also. **(4)**

Question 6 (7 marks) **(Start a New Page)** **Marks**

a) The cost of individual organ pipes of a church organ were in arithmetic progression. The smallest pipe cost \$32; the second pipe cost \$37; the third pipe cost \$42 and so on.

(i) Find the cost of the *kth* pipe **(2)**

(ii) If the **total** cost of the organ pipes was \$10443, find the number of organ pipes. **(3)**

b) For what values of x is the function $f(x) = 12x - x^2$ decreasing? **(2)**

Question 7 (8 marks) **(Start a New Page)**

a) Given that the equation **(2)**

$$3x^2 + (k-1)x + (k+4) = 0$$

has roots that are reciprocals of each other, find the value of k.

b) (i) Show why the geometric series **(1)**

$$18 + 14.4 + 11.52 + \ldots$$

has a limiting sum.

(ii) Find the limiting sum of the above series. **(2)**

c) The line $y = mx$ is a tangent to $y = x^2 - 2x + 1$. Find the possible values of m. **(3)**

END OF EXAM

Q1 a) $T_3 = 5 \times 2^{3-1}$

$= 20$

b) i) $\frac{d}{dx}(x^{\frac{1}{2}}) = \frac{1}{2}x^{-\frac{1}{2}}$

ii) $\frac{d}{dx}(4x-1)^6 = 6(4x-1)^5.4$

$= 24(4x-1)^5$

iii) $u = 2x+1 \quad v = 3x-1$

$u' = 2 \quad v' = 3$

$\frac{d(\ )}{dx} = \frac{(3x-1).2-(2x+1).3}{(3x-1)^2}$

$= \frac{6x-2-6x-3}{(3x-1)^2}$

$= \frac{-5}{(3x-1)^2}$

c) $\Delta = (3)^2 - 4(1)(k)$

$= 9-4k$

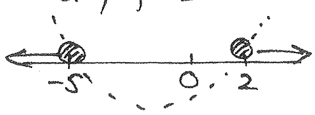
let $\Delta \geq 0$ for real roots

$9-4k \geq 0$

$9 \geq 4k$

$k \leq 2\frac{1}{4}$

Q2. a) b'dary pts $x = 2, -5$



$x \leq -5, \quad x \geq 2$

b) Let $v = x^2$

$v^2 - 3v - 54 = 0$

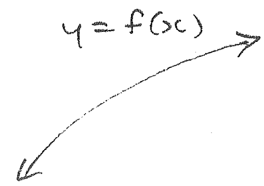
$(v-9)(v+6) = 0$

$v = 9, -6$

$\therefore x^2 = 9$ only

$x = \pm 3$

c)



d) A horizontal inflexion.

Q3 a) $(3)^2 + k(3) - 8 = 0$

$3k + 1 = 0$

$k = -\frac{1}{3}$

b) i) $\alpha + \beta = \frac{-(5)}{(2)} = -2\frac{1}{2}$

ii) $\alpha\beta = \frac{(-1)}{(2)} = -\frac{1}{2}$

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2(\alpha\beta)$

$= (-2\frac{1}{2})^2 - 2(-\frac{1}{2})$

$= 7\frac{1}{4}$

c) $\frac{dy}{dx} = 17x^{2.4} - 3$

$\frac{d^2y}{dx^2} = 40.8x^{1.4}$

Q4. a) $A(x-1)^2 + B(x-1) + C$

$= Ax^2 - 2Ax + A + Bx - B + C$

$= Ax^2 + (-2A+B)x + (A-B+C)$

$\equiv 2x^2 + 3x + 1$

$\therefore A = 2$

$-4 + B = 3$

$\therefore B = 7$

$2 - 7 + C = 1$

$\therefore C = 6$

b) $(1+5) + (2+5) + (3+5)$

$= 6 + 7 + 8$

$= 21$

c) i) $a = 5 \quad d = 3$

$T_n = 5 + (n-1)3$

$= 3n + 2$

ii) Let $T_n > 200$

$3n + 2 > 200$

$3n > 198$

$n > 66$

ie. the 67th term.

Q5. $y = x^3 - 9x^2 + 24x + 2$

i) $\frac{dy}{dx} = 3x^2 - 18x + 24$

$= 0$ for st. pts.

$3(x^2 - 6x + 8) = 0$

$3(x-2)(x-4) = 0$

$\therefore x = 2, 4$

ii) $\frac{d^2y}{dx^2} = 6x - 18$

$= 0$ for a poss. inflexn

$\therefore x = 3$

$x = 3 \rightarrow y = (3)^3 - 9(3)^2 + 24(3) + 2$

$= 20$

The inflexion is at $(3, 20)$

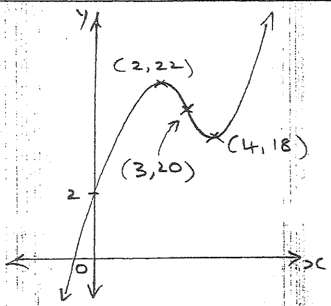
(note that because the curve is a cubic poly., and therefore continuous, there must be an inflexion)

iii)

$x = 2 \rightarrow y = 22$

$x = 4 \rightarrow y = 18$

y-intercept (when $x = 0$) is $y = 2$.



Q6 a) i) $\{32, 37, 42, \cdots\}$

is an A.P. with $a = 32$, $d = 5$

$T_n = 32 + (n-1)5$

$= 5n + 27$

$T_k = 5k + 27 \quad (\$)$

ii) $S_n = \frac{n}{2}(a + l)$

$\frac{n}{2}(32 + 5n + 27) = 10443$

$n(5n + 59) = 20886$

$5n^2 + 59n - 20886 = 0$

$n = \frac{-59 \pm \sqrt{59^2 - 4 \times 5 \times -20886}}{10}$

$= \frac{-59 \pm 649}{10}$

$= 59$ only $(n > 0)$

Hence there are 59 pipes

b) $f'(x) = 12 - 2x$

Let $f'(x) < 0$ to be decreasing

$12 - 2x < 0$

$12 < 2x$

$x > 6$

Q7 a) $\alpha\beta = 1$

$\therefore \frac{k+4}{3} = 1$

$k = -1$

b) i) $r = \frac{14.4}{18} = 0.8$

Hence $|r| < 1$

$\therefore S_\infty$ exists

ii) $S_\infty = \frac{18}{1 - 0.8}$

$= 90$

c) $\begin{cases} y = mx & \text{—— ①} \\ y = x^2 - 2x + 1 & \text{—— ②} \end{cases}$

Let ① = ②

$x^2 - 2x + 1 = mx$

$x^2 + (-2 - m)x + 1 = 0$

$\Delta = (-2-m)^2 - 4(1)(1)$

$= 4 + 4m + m^2 - 4$

$= 4m + m^2$

Let $\Delta = 0$ for tangency

$\therefore m(m + 4) = 0$

$\therefore m = 0, -4.$